How well can we predict the total cross section at the LHC?

P V Landshoff

Department of Applied Mathematics and Theoretical Physics
University of Cambridge
Cambridge CB3 0WA
England

Abstract. Independently of any theory, the possibility that the large value of the Tevatron cross section claimed by CDF is correct suggests that the total cross section at the LHC may be large. Because of the experimental and theoretical uncertainities, the best prediction is 125 ± 35 mb.

PACS: 13.85.Lg 13.85.Dz 13.60.Hb 11.55.Jy

This talk is based on work with Polkinghorne, Donnachie, Nachtmann and others going back to 1970. Further details may be found in our book[1].While theoretical understanding of long-range strong interactions has increased greatly since then, it is still not good enough to allow a confident prediction of even the value of the total cross section at the LHC. When I prepared this talk, I quoted 125 ± 25 mb, but at the meeting Alan Martin predicted 90 mb.

Alan Martin's prediction is viable only if one believes that the CDF measurement[2] of the $\bar{p}p$ cross section at the Tevatron is wrong. This is the upper of the $\sqrt{s} = 1800$ Gev data points shown in figure 1. The curves in the figure are based on ρ, ω, f_2, a_2 and soft-pomeron exchange, and they go nicely through the E710 Tevatron data point[3]. At $\sqrt{s} = 14$ TeV only the soft-pomeron term $21.7s^{0.0808}$ survives, giving a prediction of 101.5 mb.

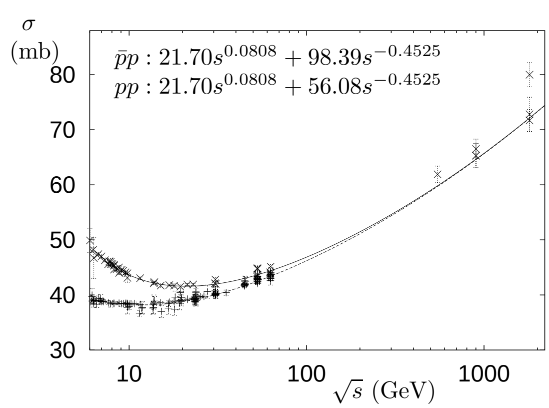

Figure 1: pp and $\bar{p}p$ total cross sections

A significant discovery at HERA was that soft-pomeron exchange does not describe the rise at small x of the proton structure function $F_2(x, Q^2)$. That is, a term that behaves

as $(1/x)^{\varepsilon_1}$ with $\varepsilon_1 \approx 0.08$ is not sufficient. As Q^2 increases, the data behave more and more as $(1/x)^{\varepsilon_0}$ with $\varepsilon_0 \approx 0.4$. The simplest description of the data at very small x is

$$F_2(x,Q^2) = f_0(Q^2)x^{-\varepsilon_0} + f_1(Q^2)x^{-\varepsilon_1} \tag{1}$$

As is well known, but is usually ignored, there are significant mathematical difficulties in the usual perturbation-theory application of DGLAP evolution at small x. Applying DGLAP evolution to a power fit such as (1) gives differential equations for the coefficient functions $f_0(Q^2)$ and $f_1(Q^2)$. However, only the one for $f_0(Q^2)$ is valid: because ε_1 is small, perturbation theory breaks down for $f_1(Q^2)$. If one extracts $f_0(Q^2)$ from fitting the small-x data, it agrees with the solution to the differential equation astonishingly well, in NLO and even in leading order[1].

So it is natural to include also a hard-pomeron-exchange term s^{ε_0} in the fits to the pp and $\bar{p}p$ total cross sections[4]. Depending on how large one makes the contribution from this term, one can make the fit go through the CDF data point, or anywhere between the CDF and E710 points. Making it go through the CDF point leads to a prediction of about 160 mb for the LHC total cross section.

This highlights the issue that is generally referred to as "unitarity", which can mean various things. One is the Froissart-Martin-Lukaszuk bound, that at large enough s

$$\sigma^{TOT} < \frac{\pi}{m_\pi^2} \log^2(s/s_0) \tag{2}$$

For reasonable values of the unknown scale s_0 this gives a bound of several barns, so it is not really relevant. A more stringent condition is obtained by writing the elastic-scattering amplitude in so-called eikonal form:

$$A(s,-\mathbf{q}^2) = 2is \int d^2b\, e^{-i\mathbf{q}.\mathbf{b}} \left(1 - e^{-\chi(s,b)}\right) \tag{3}$$

Then a constraint from unitarity is that Re $\chi(s,b) \geq 0$.

A much more difficult consequence of unitarity is that if it is possible to exchange an object such as the soft pomeron, one must also take account of the exchange of two or more of them. While we know certain general features of these further exchanges, we do not know how to make quantitative calculations. One model is to expand the exponential in (3) as a power series:

$$A(s,-\mathbf{q}^2) = 2is \int d^2b\, e^{-i\mathbf{q}.\mathbf{b}} \left(\chi - \frac{\chi^2}{2!} + \frac{\chi^3}{3!} \ldots - \frac{(-\chi)^n}{n!} \ldots\right) \tag{4}$$

and identify the first term as the contribution from the single pomeron exchange. The second term then has the correct features of the exchange of two pomerons, the third term of three, and so on. But this is only a model: there is no reason to believe that it is correct, and various reasons to believe that it is not[1].

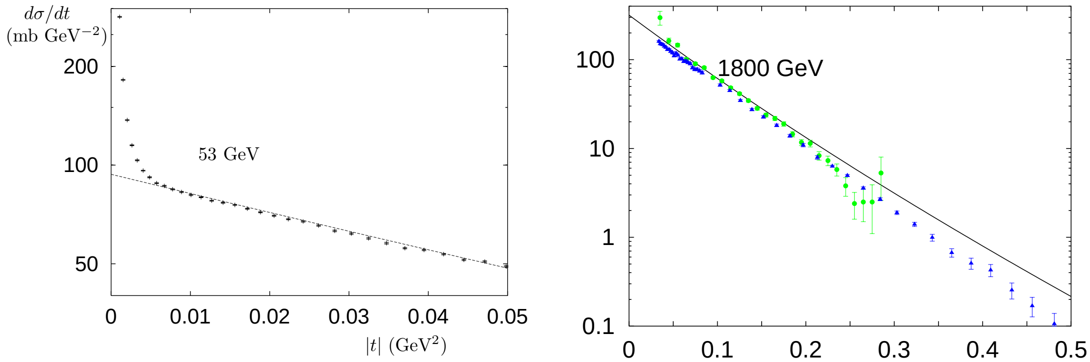

Figure 2: elastic scattering – pp at 53 GeV and $\bar{p}p$ at 1800 GeV

The left hand part of figure 2 shows that, beyond the Coulomb peak, single pomeron exchange gives an excellent fit to the pp elastic-scattering differential cross section at small and medium values of t. The right-hand part of the figure shows that, at a rather higher energy, the fit is good only at relatively small values of t. It is known that adding in the contribution from the exchange of two pomerons should bend the curve downwards. I now describe a very crude way to calculate this.

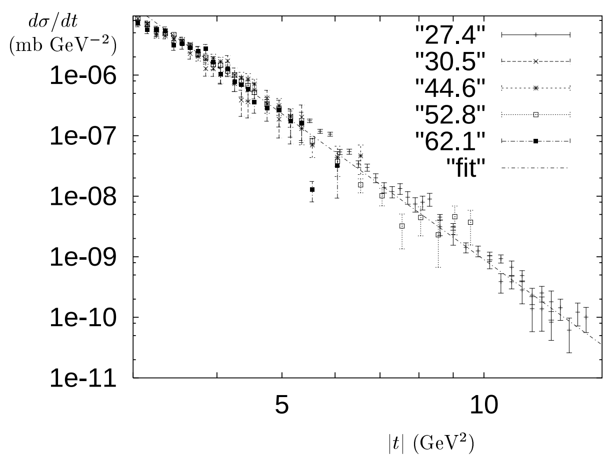

Figure 3: pp elastic scattering at large t. The line is $0.09\,t^{-8}$.

First, at large t, the data for pp elastic scattering are independent of energy and fit well to $d\sigma/dt \sim t^{-8}$. See figure 3. This behaviour is just what one gets from calculating the exchange of 3 gluons to lowest order in perturbative QCD. There is evidence that this same mechanism contributes to the creation of the dips seen in figure 4. To understand this, note first that there are rather general principles that relate the phase of an elastic amplitude to its energy dependence at that value of t. From this one knows that, near the dip, the amplitude is neither close to being real nor imaginary. This means that it is something of a coincidence that indeed there is a dip: there has to be destructive interference in both the real and the imaginary parts of the amplitude at the same value of t. The simplest way to achieve this is to cancel the imaginary parts of single-pomeron and two-pomeron exchange, and use 3-gluon exchange (which is real) to cancel the real

parts. Pomeron exchange is $C = +1$ exchange and so does not change if we replace one of the initial protons with an antiproton, but 3-gluon exchange changes sign because it is $C = -1$. So if 3-gluon exchange helps to give a dip in pp scattering, it cannot do so in $\bar{p}p$ scattering. And indeed experiment finds that $\bar{p}p$ scattering does not have a dip at $\sqrt{s} = 53$ GeV.

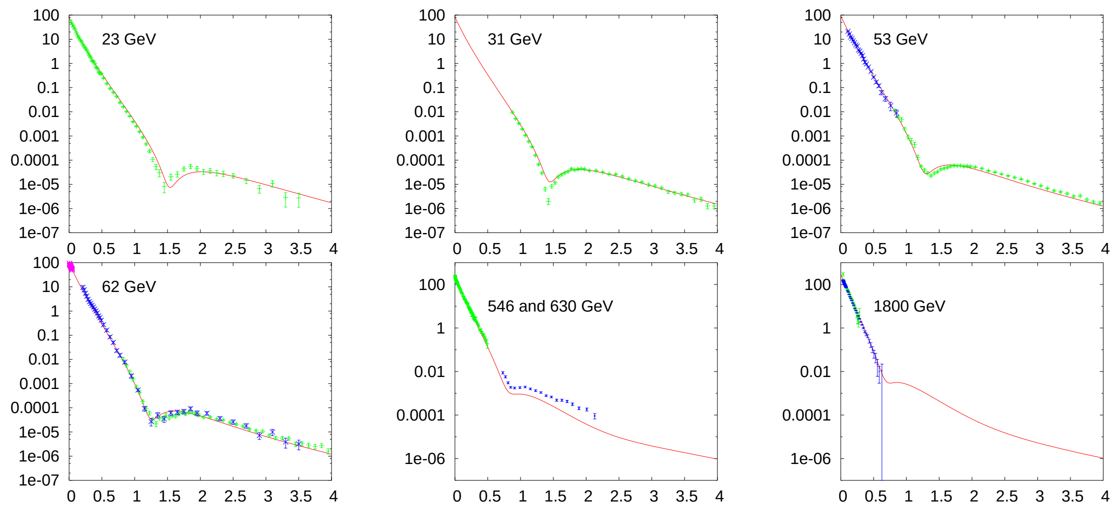

Figure 4: pp and $\bar{p}p$ elastic scattering data, with a crude model calculation.

So I have constructed a crude model, whose output is the curves in figure 4 and which is an adaptation of (4):

$$A(s, -\mathbf{q}^2) = 2is \int d^2b\, e^{-i\mathbf{q}.\mathbf{b}} \left(\chi - \frac{\lambda \chi^2}{2!} \right) \qquad (5)$$

I took $\chi(s,b)$ to correspond to the sum of the single exchanges of ρ, ω, f_2, a_2 and the soft and hard pomerons. The parameter λ determines the strength of the double exchange and is chosen so as to cancel the imaginary part of the amplitude at the dip. The 3-gluon exchange term also includes a parameter that switches off its large-t behaviour, t^{-4}, so that it does not diverge at $t = 0$.

The result is that the power behaviour of the total cross ection from single exchange is damped by the double exchange, and the extrapolation to LHC energy is pulled down from 160 mb to 125 mb. Clearly, this model is very crude, but it is the best that can be done at present.

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